Model-based identification of structural parameters using strain measurements – simulation and experiment

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ABSTRACT: Model updating methods are often used in vibration-based structural health monitoring (SHM) when a FE model is available. The approach presented here is based on the Virtual Distortion Method (VDM). The VDM was originally a method of fast reanalysis used for optimal prestress and redesign of skeletal structures in statics. With an extension to dynamics, it turned out to be a useful tool for structural parameter identification and SHM. Stiffness, mass and damping modifications are considered in this paper. Fundamental steps of the identification procedures are outlined. Good results of numerical simulations are subsequently confronted with an on-going experiment. Problems to be faced and directions of future research are mentioned.

1. INTRODUCTION

Modification of structural parameters may be interpreted as a direct change introduced to the structure but also as a change of state that the structure experiences during its lifetime. In particular such modifications may reflect structural damage. To this end, a model of the analysed structure is necessary. This model should be well calibrated i.e. its responses should be as similar to measured responses of the reference structure as possible. Then if a new set of measurements corresponding to the actual structure is available, a procedure of model updating can be started. The aim of the procedure is to determine proper modifications that need to be introduced to the model in order to match its response to current measurements. This model matching can be obtained via optimization as a sensitivity-based solution of the inverse problem e.g. as proposed by Teughels, De Roeck (2004). The model updating approaches are often used for identifying structural damage in vibration-based Structural Health Monitoring (SHM) as reported in Uhl et al. (2008).

The method of model updating proposed in this paper is called the Virtual Distortion Method (VDM). It can be classified as a method of fast reanalysis. It uses the virtual distortions as design variables in the procedure of solution of the inverse problem. These virtual distortions are pseudo quantities e.g. pseudo-strains, whose role is to modify structural response instead of introducing corrections of stiffness and mass directly as commonly performed. In order to achieve this, some pre-calculated responses of the structure, characterizing its internal relations must be known. The full set of these pre-calculated responses is called the influence matrix within the framework of VDM.

This paper describes the use of the VDM concept of model updating for identifying modifications of stiffness, mass and damping in truss structures. Considering the initial idea of virtual distortion understood as a pseudo-strain, the measured quantities in our experiments are strains, collected by piezoelectric sensors. This is quite different from the majority of approaches, which use acceleration measurements in vibration-based SHM. For the sake of speed of numerical calculations, the steady-state version of the VDM-based identification approach with harmonic excitation is presented. The numerical calculations are confronted with an experiment.

2. MODEL UPDATING BY THE VIRTUAL DISTORTION METHOD

2.1. Brief characteristic of VDM

The Virtual Distortion Method belongs to fast reanalysis methods as described by Akgun et al. (2001). This means that it needs a baseline response obtained due to an initial analysis. Subsequently this baseline response can be efficiently modified in a VDM-based reanalysis. To this end, the so-called influence matrices are needed. These matrices are sets of structural responses due to local perturbations applied either to elements or to nodes (degrees of freedom) of a structure. The local perturbations are called virtual distortions. Their application in numerical modelling is equivalent to the introduction of real modification in the structure. The number of virtual distortions which have to be considered when building the influence matrices depends upon the type of the analysed structure. The simplest case is a truss structure in which the application of one virtual distortion corresponding to the axial strain in an element is enough to describe inter-relations in structural system. For frames, bending distortion and bending plus shear distortion must be applied apart from the axial strain. For plates, the number of distortions grow and the size of influence matrices seem to limit the use of VDM. Therefore the VDM, presented in Holnicki-Szulc (2008), found its applications mostly to skeletal structures in mechanics, but also to non-structural engineering systems, which can be represented as graphs.

2.2. Identification of stiffness and mass

Modifications of stiffness can be performed both for static and dynamic analyses. The simplest way to introduce such a modification to the structure is to change Young's modulus in a truss element. This is purely a modification of material properties and therefore has a static character. The coefficient of stiffness change μ can be expressed as a ratio of the modified E' to initial E Young's modulus. This change can be modelled within the framework of the VDM by local introduction of a virtual distortion ε^0 (pseudo-strain) to the structural element which is to be modified. Thus the coefficient of stiffness change μ is expressed as:

$$\mu = \frac{E'}{E} = \frac{\varepsilon - \varepsilon^0}{\varepsilon} \tag{1}$$

Modifications of mass are inherently related to structural dynamics. In this paper, the steady state dynamics due to harmonic load is assumed, similarly to the approach described in Swiercz et al. (2008). Modifications of inertia can be numerically realized by the change of material density. A more practical approach is to change the cross-sectional area of an element, which involves both the stiffness and mass change. In order to account for the inertia modification within the framework of the VDM, another kind of virtual distortion f^0 (external pseudo-force) is needed. This pseudo-force is externally applied to nodes of a truss structure. If stiffness and mass changes are considered simultaneously and system linearity is assumed, the structural response (in terms of strains and displacements) can be expressed within the VDM as a superposition of the baseline response ^{L} and two residual responses ^{R} due to virtual distortions ε^0 and f^0 respectively:

$$\begin{aligned} \varepsilon &= \varepsilon^L + \varepsilon^R = \varepsilon^L + D^\varepsilon \varepsilon^0 + D^f f^0 \\ u &= u^L + u^R = u^L + B^\varepsilon \varepsilon^0 + B^f f^0 \end{aligned} \tag{2}$$

where: ε – the total strain, ε^0 – the virtual distortion modelling stiffness modifications, f^0 – the virtual distortion modelling mass modifications, D^ε, D^f, B^ε, B^f – the respective influence matrices e.g. D^ε stores strain responses due to unit strain distortions.

The equation responsible for mass change modelling can be obtained by comparing two equations of motion for the structures with actual and virtual (by f^0) modifications. After rearrangements the relation for harmonic motion at frequency ω (with quasi-static amplitudes of f^0 only) yields:

$$-\omega^2 \Delta M u + f^0 = 0 \tag{3}$$

where: ΔM – the global matrix of mass modifications.

If both damage degradation and mass loss are considered, eq. (1) and (3) constitute a set of equations which should be solved for the stiffness-modelling distortions ε^0 and mass-modelling distortions f^0.

In the identification process, a minimum of the following objective function is sought:

$$F(\mu) = \left(\frac{\varepsilon - \varepsilon^M}{\varepsilon^M}\right)^2 \tag{4}$$

where: ε^M – the measured strain.

The modification coefficient has to be bounded in the range <0,1> in order to consider structural damage. Gradient of the function (4) with respect to the design variable μ is calculated using the chain rule of differentiation. Partial derivatives of ε^0 and f^0 with respect to μ are computed by both side differentiation of the set of equations (1) and (3), which produces the same left-hand side matrix as in the primary set (1) and (3). The design variable μ is updated in iterations by an optimization routine e.g. simple steepest descent as in Kolakowski et al. (2004) or advanced Levenberg-Marquardt as in Kolakowski et al. (2008).

The described model updating process effectively solves an inverse problem of parameter identification relying on strain measurements and gradient-based optimization methods. It is important to notice that the same optimization procedure can be used to calibrate the numerical model to experiment, which is a *sine qua non* condition for subsequent successful identification of stiffness and mass modifications.

2.3. Identification of damping

Damping is another parameter that can be identified by the VDM. The proposed approach is used to solve the inverse problem of identification of material damping by decomposing it into two linear subproblems. Confining the considerations to material damping only, it is assumed that the damping parameters can be identified for each member of a truss structure independently. Thus the damping matrix C can be expressed as:

$$C = G^T L S C^\beta G \tag{5}$$

where: $C^\beta = \mathrm{diag}\{\beta_1, \dots, \beta_{N\_el}\}$, $K = G^T LSG$ is the stiffness matrix, L is the diagonal matrix of element lengths, G is the displacement-strain matrix, which relates the global displacements u to local element strains ε, S is the diagonal matrix of element stiffnesses $E_i A_i$.

The steady state response of the structure under harmonic excitation is considered, therefore the exciting force of the complex exponential form is applied to the model. The motion of the structure with only material damping modelled by virtual distortions φ^0 is described by the equation expressing respective amplitudes:

$$-\omega^2 Mu + G^T LS\left(i\omega C^\beta \varepsilon - \phi^0\right) + G^T LS\varepsilon = f \tag{6}$$

Assuming that the member forces in the modified and modelled (by φ^0) structures are equal, the relation between the distortion and the equivalent modification of material damping can be found as:

$$-i\omega\eta\varepsilon = \phi^0 \tag{7}$$

where η is defined as $\eta = \hat{S}\hat{C}^\beta - SC^\beta = (\mu - 1)SC^\beta$ and symbols with ^ refer to modified structural parameters.

The identification procedure consists of two steps. The first one is to find the virtual distortions that correspond to the measured strain response ε:

$$D^\varepsilon \phi^0 = \varepsilon - \varepsilon^L \tag{8}$$

Since in statically indeterminate structures the influence matrix D^ε is singular, there is an infinite number of distortions φ^0 that solve equation (8) in the least-squares sense. These distortions form a linear subspace that can be computed using the Singular Value Decomposition (SVD) of the influence matrix. The equivalent virtual distortions can be represented in the following form:

$$\phi^0 = \phi^{SVD} + Wx \tag{9}$$

where the columns of the matrix W generate the null space of D^{ε}, the vector x contains arbitrary complex numbers and ϕ^{SVD} is the least-squares solution of equation (8).

The second step is to compute the material damping coefficients using eq. (7). Since the strain response ε of the truss structure is measured in each element, the eq. (7) become decoupled and form a set of linear equations, with one unknown per equation. Additional assumption is that η must be real numbers. By dividing complex quantities into real and imaginary parts, a set of real equations can be obtained from which the coefficients η may be calculated:

$$\begin{bmatrix} \operatorname{Re} W & -\operatorname{Im} W & -\omega \operatorname{diag} \operatorname{Im} \varepsilon \\ \operatorname{Im} W & \operatorname{Re} W & \omega \operatorname{diag} \operatorname{Re} \varepsilon \end{bmatrix} \begin{bmatrix} \operatorname{Re} x \\ \operatorname{Im} x \\ \eta \end{bmatrix} = \begin{bmatrix} -\operatorname{Re} \phi^{SVD} \\ -\operatorname{Im} \phi^{SVD} \end{bmatrix} \tag{10}$$

3. NUMERICAL SIMULATIONS VS. EXPERIMENT

3.1. Considered structure

The VDM-based parameter identification procedure is demonstrated for a 2D fifteen-element truss structure, depicted in Fig. 1a, made of steel (E = 204 GPa). A view of the corresponding numerical model with element numbers is shown in Fig. 1b. Two top nodes of the truss are fully suppressed. The structure consists of three sections 0.75 m x 0.4 m each. The total length of the structure is 2.25 m. Twelve elements have uniform cross-sectional areas A = 0.56 cm^2. Three diagonals (no. 4, 10, 14) are double elements of cross-sectional areas 2A = 1.12 cm^2 with a gap between them to let the cross-diagonals in. Concentrated masses of 0.592 kg and 0.639 kg were added in the model in bottom and middle nodes respectively in order to account for the additional mass of joints in the real structure.

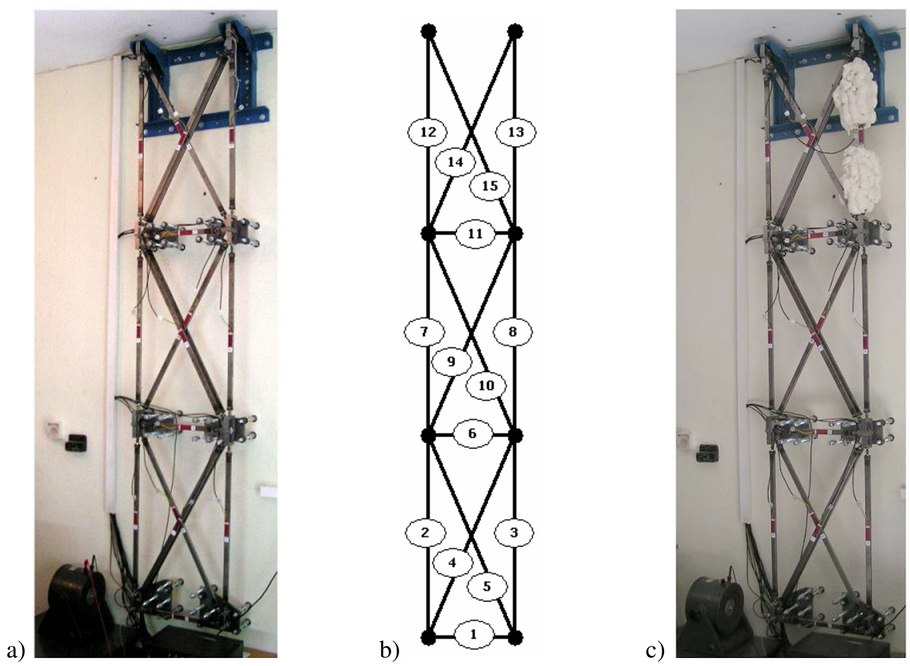

Fig. 1 Analyzed structure – a) the reference truss, b) its numerical model, c) the truss with construction foam

3.2. Numerical simulations of stiffness/mass modifications

A scenario of stiffness/mass modifications was assumed with the following coefficients of cross-sectional modifications: μ_3=0.7, μ_7=0.5, μ_{14}=0.8 in elements no. 3, 7 and 14 respectively. The result of parameter identification for pure data can be seen in Fig. 2. The influence of 5% noise can be analyzed in Fig. 3. In general, good quality of identification is observed.

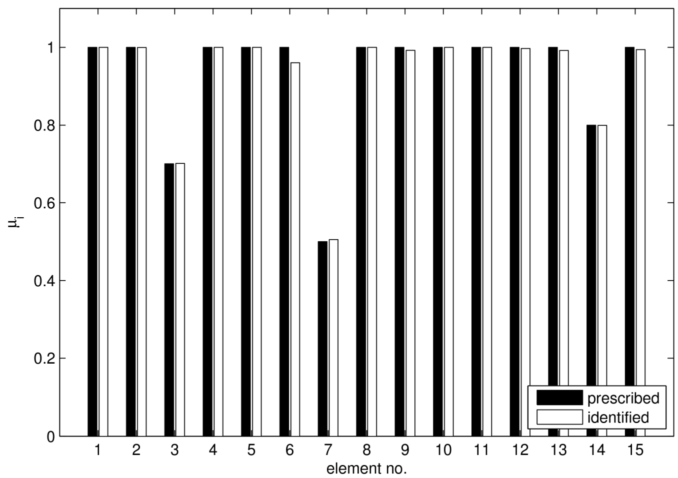

Fig. 2 Numerical identification of stiffness/mass parameter for assumed scenario.

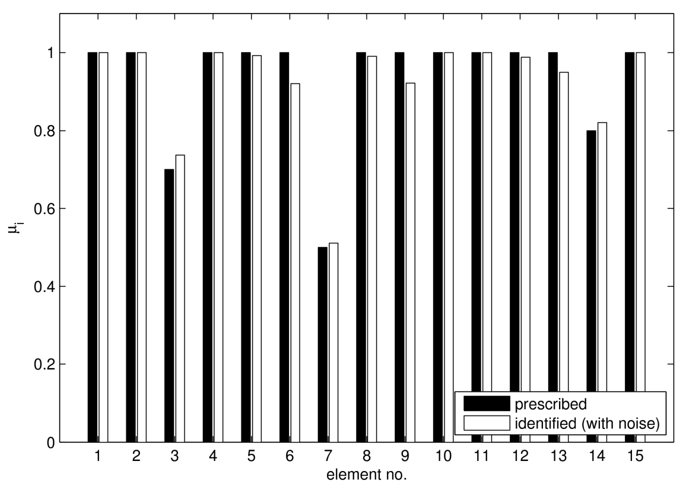

Fig. 3 Numerical identification of stiffness/mass parameter for assumed scenario (with 5% noise).

3.3. Numerical simulations of damping modifications

The damping matrix was initially assumed using the classical Rayleigh model with coefficients which correspond to 1% critical damping. Based on the proposed model of the material damping, the actual damping (introduced modifications) is assumed to increase three μ_5=3, five μ_8=5 and two times μ_9=2 in elements no. 5, 8 and 9 respectively. Figure 4 shows the comparison of η (defined in 2.3) obtained from the identification procedure vs. the prescribed values.

In order to check the stability of the identification method from the practical point of view, the simulated measured response was contaminated with uncorrelated Gaussian noise at 5% RMS level of ε-ε^L. Figure 5 shows the comparison of η obtained from identification procedure with additional noise vs. the prescribed values. Elements 1, 6, 11 are not depicted in the comparison because strains in these horizontal elements are very small and the applied noise influences member responses significantly.

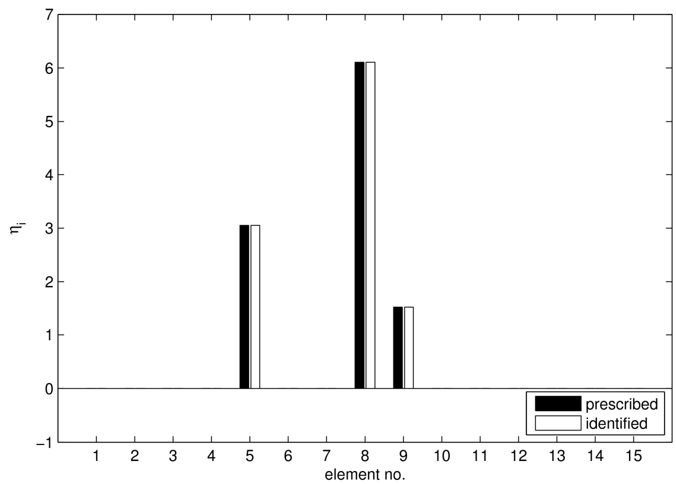

Fig. 4 Numerical identification of damping parameter for assumed scenario.

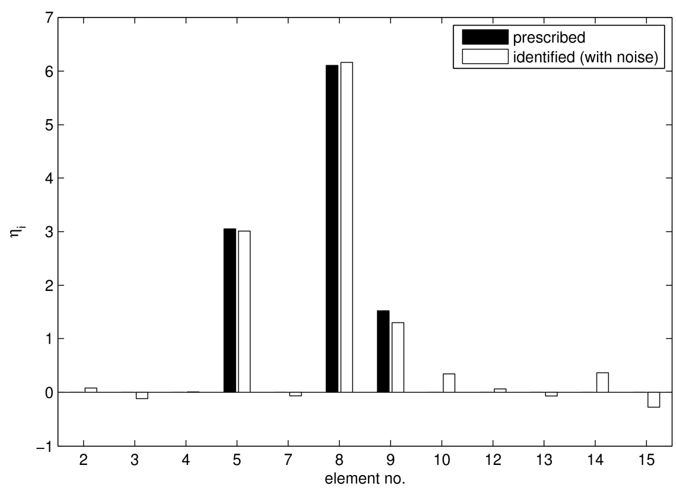

Fig. 5 Numerical identification of damping parameter for assumed scenario (with 5% noise).

3.4. Reference measurements and tuning the numerical model

The structure was excited with a harmonic force of 70 N amplitude at the frequency 19 Hz, applied horizontally to the left bottom node via a modal shaker. Each element of the truss was equipped with a piezoelectric fibre composite (PFC) sensor collecting strains. The measurement data were acquired by a National Instruments card and processed by the LabView software.

The experiments were carried out for the reference structure first in order to match the numerical model to measurements. Due to various imperfections of the real truss set-up e.g. not ideal application of load through a stinger, possible friction at nodes, etc., it turned out that the horizontal elements 1, 6 and 11 respond at much higher level in the experiment than in numerical computations. Also the model-measurements discrepancy in elements 2, 5, 9 and 15 is big. Therefore the seven mentioned elements were not taken into account when matching the responses of the numerical model and the real structure. The procedure of stiffness/mass identification, described in 2.2, was run to tune the model. The result of the tuning is depicted in Fig. 6.

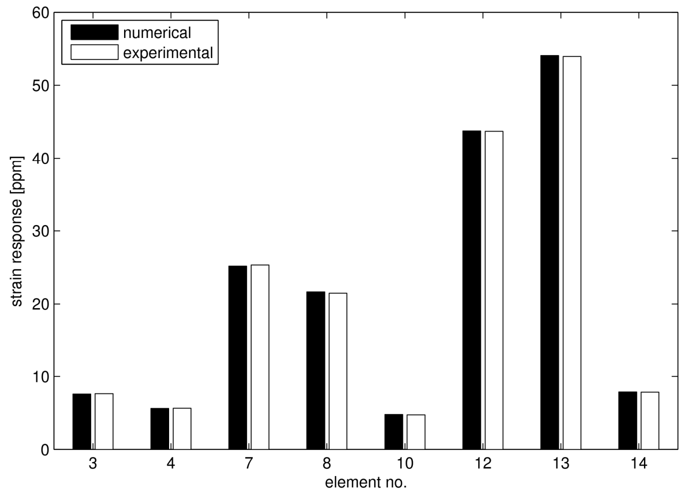

Fig. 6 Result of matching the numerical model to experimental data for eight elements.

3.5. Comparison of the reference and modified responses in experiment

A scenario of parameter modification was realized by applying construction foam to the near-support element no. 13 as shown in Fig. 1c. The mass of the foam was approximately 70% of the mass of element 13. The influence of this modification in all elements can be studied in Fig. 7. The presented responses (original and with foam) are an average of six measurements.

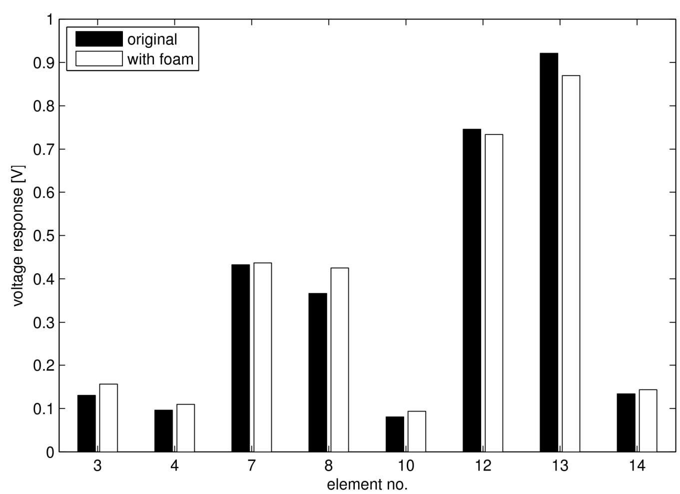

Fig. 7 Comparison of structural responses for the reference and modified case in experiment.

It can be seen that application of the foam does not make the structure respond very differently. The most significant change is in elements 8 and 13.

It was observed that the energy of excitation at 19 Hz was quite significant. Unfortunately some of it was apparently dissipated through the base, on which the shaker was standing, due to its slight rocking on uneven floor. Therefore it was decided to improve the excitation conditions before testing the parameter identification procedure using experimental data.

4. CONCLUSIONS

An approach to structural parameter identification based on the Virtual Distortion Method (VDM) is presented. The method was applied to skeletal structures, in particular trusses. The procedure of model updating by VDM consists in modifying structural properties by introducing pseudo quantities to the system i.e. pseudo-strains or external pseudo-forces, generally called virtual distortions. Three kinds of modifications are considered: stiffness, mass and damping. The results of numerical simulations using VDM-based algorithms are very good even with the presence of simulated noise. An experiment was conducted in order to verify the simulation results. First, the numerical model of the investigated structure was matched to experimental responses. A modification scenario with construction foam added on one element was considered. Unfortunately it turned out that this modification did not make the structure respond very differently. It is expected that an improvement of the excitation conditions should help to transfer the applied force to the structure more efficiently, which is supposed to influence the quality of responses.

Further work will be focused on analyzing more scenarios of parameter modifications, e.g. replacement of elements with different cross-sectional areas, in particular several modifications introduced simultaneously. A generalization of the identification procedure allowing not only the harmonic but also a time-varying excitation signal will be worked upon.

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